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INVESTIGATION AND CALCULATION OF THE STRESSED-STRAINED STATE AND COMPACTION OF VISCOELASTIC DISPERSE MEDIA AS A RESULT OF RELAXATION PROCESSES

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A theoretical study of the change in the stressed-strained state and compaction, under loading, of viscoelastic disperse media (including soils) with rheological properties varying with deformable-layer depth has been made. The processes of creep and relaxation of such media have been investigated. Calculations from the formulas derived for compactible soils have been carried out based on experimental data on the rheological properties of soils.

High compressibility, which is mainly due to the process of compaction under the action of external load, is one distinguishing property of viscoelastic disperse media. Among such media are binder loose soils with humidities w lower than the total moisture capacity and grounds [1–3].

To model the properties of viscoelastic disperse media one can use results obtained in the mechanics of a deformable rigid body [4–7]. Data on studying the processes of creep, stress relaxation, and change in the stressed-strained state of deformable media and materials under different loading conditions are available in [5–7]. In these works, the initial physical properties of a medium are taken to be constant at different depths of the deformable layer. In the present investigation, we consider the cases of strain of a disperse medium with its density and rheological properties varying with deformable-layer depth. The results obtained refer to different disperse compactible media. However, soils are the main object of the investigation carried out; we give experimental and calculated data for soils in this work.

Based on a theoretical analysis of the regularities (revealed experimentally) of strain of compactible soils [2, 3], it has been proposed [8–10] that their rheological properties be modeled by the linear Volterra integral equation of the second kind with a nonlinear free term and Koltunov's kernel quite accurately describing the regularities of compressive and shear strain of such soils with time in a wide range of variation of their density and humidity. It has been shown that for certain values of t, the compressive stresses σ, and the relative compressive strains ε this governing integral equation of a nonlinear hereditary viscoelasticity theory can approximately be replaced by the differential equation

$$\sigma_t' + p\sigma = q\varepsilon_t' . \qquad (1)$$

Based on a theoretical analysis of the strain properties of soils, which were revealed experimentally [2, 3, 9], it has been proposed that this equation be used as a governing one for soils [11]. The parameters p and q comprehensively characterize the rheological properties of soil without separating them into elastic and viscous components.

The range of application of Eq. (1) to specific soils is revealed experimentally. Its suitability for derno-podzolic light loamy soil of a certain granulometric composition for w = 16–26% and certain values of t, σ, and ε [8–11] and black earth soils of the mid-Volga region has been confirmed [12]. The advantages of application of Eq. (1) have been shown. In straining by the harmonic law, we have $p = \omega g$. For the derno-podzolic soil investigated (for $\rho_{dr} = \rho/(1 + 0.01w)$ = 1.138–1.579 g/cm^3, w = 16–26%, and ω = 0.93–5.01 sec^{-1}), we have found the following linear regression equations [9, 10]:

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$$g = 14.655 - 6.716\rho - 0.581\omega + 0.085w\ , \tag{2}$$

$$q = 14.981\rho + 0.245\omega - 0.315w - 9.654\ . \tag{3}$$

For the characteristic of the viscoelastic properties of the black earth soil, we have obtained [12] the regression equations

$$p = 2.8 - 1.4\rho + 0.5\omega\ , \tag{4}$$

$$q = 2.46\rho - 1.42\ . \tag{5}$$

Based on the results of [10], we have investigated and mathematically modeled relaxation processes in soils whose properties are described by Eq. (1). It is assumed that the deformable soil layer propagated to depth H is located on a rigid base, for example, on a soil layer whose strains are negligible. The surfaces of the deformable soil layer and the rigid base are horizontal. We introduce a fixed rectangular coordinate system $Oxzy$ with its origin at the point O located on the soil surface; the x axis is directed horizontally to the right, and the y axis is directed vertically downward.

In accordance with the results of statistical processing of experimental data, we take the linear dependence of the density ρ of a deformable soil layer before the action of external load on it on the depth y:

$$\rho\ (y) = \overset{\frown}{\rho} + ky\ . \tag{6}$$

The external load is transferred to the soil surface through a stamp. The stamp width along the z axis is taken to be fairly large; in this case the strain of the soil may be assumed to be plane. Compressive strains of the soil in the vertical direction y and shear strains in the horizontal direction occur in the Oxy plane under the action of an external load. The change in the density of the soil is mainly caused by its vertical displacement; therefore, we did not consider shear strains.

We model the process of propagation of viscoelastic damping waves of compressive strain of soil in a compactible soil under its loading. The formulas and algorithms of solutions enabling us to find the depth H_{pr} of propagation of the compressive strain of the soil, the increment of the density $\Delta\rho(y)$, and the density of the soil at different depths have been obtained [9]. In the investigation whose results are given below, we have considered the case where the value of H is low; here, $H_{pr} = H$. The soil at the depth H was taken to have the maximum possible density before the action of an external load on it; therefore, we had $\Delta\rho(H) = 0$. The relative compressive strain of the soil was $\varepsilon(t) = h(t)/(H - h(t))$.

In viscoelasticity theory, one more frequently considers relaxation processes in deformable media at constant levels of stresses or strains resulting from step loading, i.e., each specified stress level is taken to be reached instantaneously [5–7]. In this work, we have taken into account that the external load increases over a period of time and does not act instantaneously. We have investigated the creep and relaxation of soils that result from their loading in short time intervals, namely, in seconds and their fractions.

To investigate the process of relaxation we have considered two stages of change in the stressed-strained state of a compactible soil. At the first (initial) stage, the strain of the soil and the stresses in it change at $t \in [0; t_0]$ (they increase from the zero values to ε_0 and σ_0). At the second stage (of relaxation), the relative strain is $\varepsilon = \varepsilon_0 = \text{const}$ at $t \in [t_0, \infty]$, and the stresses change with time: $\sigma = \sigma(t)$. We investigated the relaxation processes occurring after the initial strain of the soil by the harmonic and linear laws.

Let the change in the relative compressive strains of the soil at the first stage follow the harmonic law

$$\varepsilon\ (t) = \varepsilon_m \sin \omega t\ , \quad t \in [0; t_0]\ . \tag{7}$$

Substituting ε_t' into (1), with account for the initial condition (stress $\sigma = 0$ at $t = 0$), we obtain

$$\sigma(t) = \frac{q\varepsilon_m}{g^2+1} (\sin \omega t + g \cos \omega t - g \exp(-g\omega t)) , \quad t \in [0; t_0] . \tag{8}$$

The numerical values of the quantities g and q involved in (8) correspond to $\rho = \hat{\rho}$.

The strain of the soil, the depth of propagation of its deformable layer, and the density change with t as a result of the action of the external load. Before the beginning of the relaxation stage, we have $h_0 = h(t_0)$ and $H_0 = H - h_0$.

At the second stage of change in the stressed-strained state of the soil, we take the coordinate $y = h_0$ as a new reference point of the depth $\tilde{y}$ (i.e., as the medium's surface). At the beginning of the relaxation, the density of the soil for $\tilde{y} = 0$ is equal to

$$\hat{\rho}_0 = \hat{\rho} + kh_0 + \Delta\rho (h_0) . \tag{9}$$

In accordance with [9], the dependence of the density of the soil on $\tilde{y}$ can be taken to be linear with a high degree of accuracy. Knowing $\hat{\rho}_0$ and $\rho(H_0)$, we find the dependence of the form (6) of the soil density on $\tilde{y}$. Here, in Eq. (6), we have $y = \tilde{y}$, $\rho(y) = \rho_0(\tilde{y})$, $\hat{\rho} = \hat{\rho}_0$, and $k = k_0$.

The relaxation properties of the soil, which are determined by the new values g_0 and q_0 of the characteristics of its viscoelastic properties corresponding to the equality $\hat{\rho} = \hat{\rho}_0$ at the beginning of the relaxation stage, change with soil density. Since g and q linearly depend on the soil density, their new values are also linear functions of y. At $t \in [0; \infty]$, the characteristics of the viscoelastic properties of the soil are linear functions of $\tilde{y}$.

At the relaxation stage, we have $\varepsilon = \varepsilon_0 = \varepsilon_m \sin \omega t_0 = \text{const}$. Substituting $\varepsilon_t' = 0$ into (1) and taking the initial condition (stress $\sigma = \sigma_0$ at $t = t_0$) into account, we obtain the formula characterizing the process of relaxation:

$$\sigma(t) = \sigma_0 \exp(-g_0\omega t) , \quad t \in (t_0, \infty) . \tag{10}$$

The quantity σ_0 involved in (10) is determined from (8) at $t = t_0$. From formula (10) it follows that $\sigma(t) \to 0$, if $t \to \infty$.

To investigate the process of creep we have considered two stages of change in the stressed-strained state of the soil, just as in investigating the process of relaxation. The strains in the soil and the stresses in it change, increasing from the zero values to ε_0 and σ_0, at the first (initial stage) at $t \in [0; t_0]$. At the second stage (of creep), we have the stresses $\sigma = \sigma_0 = \text{const}$ at $t \in [t_0; \infty]$, and the relative compressive strain of the soil changes with time: $\varepsilon = \varepsilon(t)$. We investigated the creep processes occurring after the initial strain of the soil by the harmonic and linear laws.

Let the change in compressive stresses at the first stage follow the harmonic law:

$$\sigma(t) = \sigma_m \sin \omega t , \quad t \in [0; t_0]. \tag{11}$$

Substituting σ_t' into (1), with account for the initial condition (strain $\varepsilon = 0$ at $t = 0$) we obtain

$$\varepsilon(t) = \frac{\sigma_m}{q} (\sin \omega t - g \cos \omega t + g) , \quad t \in [0; t_0] . \tag{12}$$

The numerical values of the quantities g and q involved in (12) correspond to $\rho = \hat{\rho}$.

The strain of the soil, the depth of propagation of its deformable layer, and the density change with t as a result of the action of the external load. Before the beginning of the creep stage, we have $h_0 = h(t_0)$ and $H_0 = H - h_0$.

At the second stage of change in the stressed-strained state of the soil, we take the coordinate $y = h_0$ as a new reference point of the depth $\tilde{y}$. At the beginning of the creep, the soil density for $\tilde{y} = 0$ is obtained from formula (9). Knowing $\hat{\rho}_0$ and $\rho(H_0)$, we find the dependence of the form (6) of the soil density on $\tilde{y}$. The characteristics g_0 and q_0 of the viscoelastic properties of the soil at the beginning of the creep stage correspond to the value $\rho = \hat{\rho}_0$. At the creep stage, we have $\sigma = \sigma_0 = \sigma_m \sin \omega t_0 = \text{const}$. Substituting $\sigma_t' = 0$ into (1) and taking the initial condition (strain $\varepsilon = \varepsilon_0$ at $t = t_0$) into account, we obtain a formula characterizing the process of creep of the soil:

$$\varepsilon(t) = \varepsilon_0 + \frac{g_0 \omega}{q_0} \sigma_0 t\,, \quad t \in (t_0, \infty)\,. \tag{13}$$

The quantity ε_0 involved in (13) is determined from (12) at $t = t_0$.

When the values of the parameters are constant, Eq. (13) describes linear creep, which is characteristic of a Maxwell ideal viscoelastic medium and not of an actual soil. Its strain becomes stabilized with time for $\sigma_0 < \sigma_{\text{str}}$.

The compressive strains of the soil, the depth of a deformable layer of the soil, and the characteristics g and q of its viscoelastic properties changing in the process of creep until the strain becomes stabilized are continuous functions of time. They are approximately taken to be constant in short time intervals; their change is discrete.

An algorithm of determination in short time intervals $\Delta t_s = t_s - t_{s-1}$ ($s = 1, 2, ..., N$) of the increments of the relative $\Delta\varepsilon_s$ and absolute Δh_s compressive strains of the soil, the increment of the soil density at different depths, the characteristics q_s and g_s of the viscoelastic properties of the soil, the total settling of the soil h_s at $t = t_{s-1}$, the depth of the deformable soil layer H_s, and the indices of the stressed-strained state of the soil at different fixed instants of time has been developed.

The characteristic q_s increases, whereas g_s decreases with time; in this case $\Delta\varepsilon_s \to 0$, $\Delta h_s \to 0$, and $h_s \to h_{\text{st}} = \text{const}$. The time in which h_s attains h_{st} represents the time t_{st} of stabilization of the strain. As a result of the calculations we obtain: the increment of the settling $\Delta h_N \approx 0$ at $\Delta t_s = \Delta t_N$, the total settling of the soil $h_N \approx h_{\text{st}}$, and $t_N \approx t_{\text{st}}$.

We have developed computer programs enabling us to find, using the obtained formulas and computational algorithms, the indices characterizing the stressed-strained state and the density of soil at different fixed instants of time in the process of its loading at $t \in [0;\, t_0]$ and in the processes of creep and relaxation. According to these programs, we have calculated the above indices for derno-podzolic light loamy soil of known granulometric composition with a humidity of $\omega = 16$–24% (with the characteristics of viscoelastic properties, determined from formulas (2) and (3)) and for black earth soil (with the parameters of Eq. (1), found from formulas (4) and (5)). We took $H = 0.51$ m in the calculations.

We investigated the influence of the initial density and humidity of the soil, the velocities v and w, and the time t_0 on its creep, relaxation, and compaction. To reveal the character and to quantitatively evaluate the influence of these factors we performed a number of computational runs (computer experiments). Single-factor and full-scale three-factor computer experiments were carried out.

In investigating the process of relaxation, we found $\sigma(t)$ at different fixed instants of time $t_s \in [0;\, t_N]$, $\Delta\rho(h_0)$, and $\rho_0(0.05)$ at $t = t_0$. The instants of time corresponding to the values $\sigma_N = \sigma(t_N) \le 10^{-4}$ kPa, $t_N \approx t_{\text{st}}$, were taken as the approximate values of t_{st}.

In investigating the process of creep, we determined $\Delta\varepsilon_s$ and Δh_s in the time intervals Δt_s, the total values of ε_s and h_s, the velocities $(\varepsilon_s)_t'$ and $(h_s)_t'$ at different fixed instants of time $t_s \in [0;\, t_N]$ $(t_N \approx t_{\text{st}})$, the corresponding values of $\Delta\rho(\Delta h_s)$ and the density $\overset{\frown}{\rho}_0(0.05)$ of the soil, the parameters of the linear dependence of the soil density on the depth, and the characteristics of the viscoelastic properties of the soil. The instants of time corresponding to the values $\Delta\varepsilon_s \le 10^{-5}$ were taken as the approximate values of t_{st}.

We carried out eight runs of single-factor experiments (numerical computer experiments) with the aim of investigating the relaxation and ten runs of single-factor experiments to investigate the creep.

The experiments on investigating the relaxation were performed for two regimes of the initial strain of the soil at $t \in [0;\, t_0]$: by the harmonic law (7) at $t_0 = 0.2$ sec and $\omega = 3.15$ sec^{-1} and by the linear law $\varepsilon = vt$ ($v > 0 = \text{const}$) at $t_0 = 0.01$ sec and $v = 1.2$ m/sec. The experiments on investigating the creep of the soil were carried out for two regimes of change in the stresses at $t \in [0;\, t_0]$: by the harmonic law (12) at $t_0 = 0.1$ sec and $\omega = 3.15\ ^{-1}$ and by the linear law $\sigma = vt$ ($v > 0 = \text{const}$) at $t_0 = 0.1$ sec and $v = 1.2$ m/sec. In each run of single-factor experiments, we varied one of the four basic influencing factors: 1) the initial density of the soil; 2) the frequency ω or the linear rate v of change in the strains or stresses at $t \in [0;\, t_0]$; 3) the humidity w of the soil; 4) the time t_0.

The significant influence of the quantities ρ, w, and ω or v and t_0 on $\sigma(t)$ and $\varepsilon(t)$ and on the other characteristics was revealed.

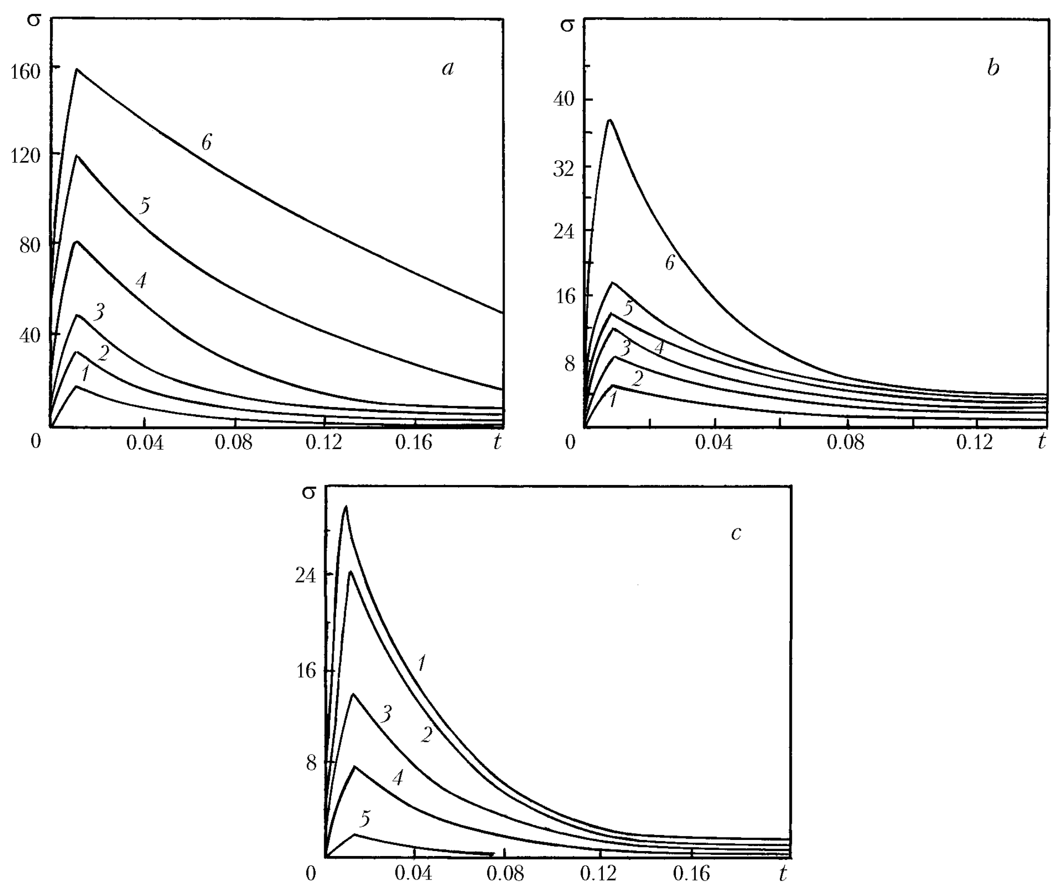

Fig. 1. Change in the compressive stresses of derno-podzolic light loamy soil with time under its initial strain by the linear law and their relaxation for different values of: a) the initial density of the soil [1) $\hat{\rho}$ = 1.1 and k = 1.7646, 2) 1.2 and 1.5686, 3) 1.3 and 1.3725, 4) 1.5 and 0.9804, 5) 1.7 and 0.5882, and 6) 1.9 g/cm^3 and 0.1960 g/(cm^3·m), v = 1.2 m/sec and w = 19%]; b) the rate of initial strain [1) v = 0.5, 2) 0.7, 3) 0.9, 4) 1.0, 5) 1.2, and 6) 2 m/sec, $\hat{\rho}$ = 1.1 g/cm^3, k = 1.7646 g/(cm^3·m), and w = 19%]; c) the humidity of the soil [1) w = 16, 2) 17, 3) 20, 4) 22, and 5) 24%, $\hat{\rho}$ = 1.1 g/cm^3, k = 1.7646 g/(cm^3·m), v = 1.2 m/sec, and w = 19%]. σ = kPa; t, sec.

Figure 1 gives the plots reflecting the results of certain experiments on investigating the stress relaxation in the derno-podzolic soil; these plots have been obtained for the case of change in $\varepsilon(t)$ by the linear law $\varepsilon = vt$ at $t \in [0; t_0]$. Also, we have constructed the curves characterizing the dependences $\sigma(t)$ for the case of change in $\varepsilon(t)$ by the law (7) at $t \in [0; t_0]$. These plots characterize the processes of buildup of the stresses at $t \in [0; t_0]$ and their relaxation at $t \in [0; \infty]$.

The dependences $\sigma(t)$ obtained show that higher values of the initial density of the soil correspond to higher values of the stresses σ_0 for the same ε_0. In a loose soil, the stresses $\sigma(t)$ are smaller; they relax and tend to zero faster than those in a denser soil; the time t_{st} increases with soil density. Higher values of the strain rate correspond to higher values of ε_0 and σ_0 and to a shorter time of relaxation of the stresses in the soil. As the deformation rate increases in the period $t \in [0; t_0]$, the compaction of the soil also increases. A change in the humidity of the soil causes its relaxation properties to change. As w grows, the stresses $\sigma(t)$ decrease and relax faster at all values of t.

The time t_0 also substantially influences the compaction of the soil and the stress relaxation. In the case of change in $\varepsilon(t)$ by the law (7), we determined the indices, varying t_0 from 0.05 to 0.45 sec, whereas in the case of change in $\varepsilon(t)$ by the linear law we varied t_0 from 0.005 to 0.1 sec. The calculations have shown that the quantities

TABLE 1. Compaction, Initial Strains, and Creep of Derno-Podzolic Soil for Different Values of Its Initial Density

Experiment No.	Parameters of dependence (6)		ρ_0, g/cm^3	k_0, g/(cm^3·m)	σ_0, kPa	ε_0	$\varepsilon(1)$	$\varepsilon(20)$	$\hat{\rho}_0(0.05)$, g/cm^3 at t = 10 sec
	$\hat{\rho}$, g/cm^3	k, g/(cm^3·m)							
1	1.1	1.7647	$\frac{1.1435}{1.1005}$	$\frac{1.7032}{1.7640}$	$\frac{10.8}{0.12}$	$\frac{0.014}{10^{-4}}$	$\frac{0.0176}{0.130}$	$\frac{0.236}{0.240}$	$\frac{1.8386}{1.8532}$
2	1.2	1.5686	$\frac{1.2223}{1.2002}$	$\frac{1.5356}{1.5682}$	$\frac{10.8}{0.12}$	$\frac{0.007}{10^{-4}}$	$\frac{0.044}{0.046}$	$\frac{0.207}{0.211}$	$\frac{1.8400}{1.8640}$
3	1.3	1.3725	$\frac{1.3147}{1.3002}$	$\frac{1.3496}{1.3722}$	$\frac{10.8}{0.12}$	$\frac{0.004}{10^{-5}}$	$\frac{0.031}{0.004}$	$\frac{0.154}{0.167}$	$\frac{1.8600}{1.9000}$
4	1.5	0.9804	$\frac{1.5986}{1.5001}$	$\frac{0.9661}{0.9882}$	$\frac{10.8}{0.12}$	$\frac{0.002}{10^{-5}}$	$\frac{0.020}{0.002}$	$\frac{0.098}{0.126}$	$\frac{1.8850}{1.9131}$
5	1.7	0.5882	$\frac{1.7056}{1.7001}$	$\frac{0.5781}{0.5881}$	$\frac{10.8}{0.12}$	$\frac{0.001}{10^{-5}}$	$\frac{0.010}{0.001}$	$\frac{0.060}{0.090}$	$\frac{1.9411}{1.9512}$
6	1.9	0.1966	$\frac{1.9040}{1.9000}$	$\frac{1.1885}{0.1959}$	$\frac{10.8}{0.12}$	$\frac{0.001}{10^{-5}}$	$\frac{0.005}{10^{-4}}$	$\frac{0.003}{0.040}$	$\frac{1.9849}{1.9863}$

Note. Above the line, in initial loading by the law (10); below the line, by the linear law $\sigma = vt$.

σ_0, ε_0, h_0, and $\Delta\rho(h_0)$ increase with t_0; the time during which they become virtually equal to zero increases in the process of relaxation of the stresses.

Table 1 gives data reflecting the results of certain experiments on investigating the creep of derno-podzolic soil. Here we give the numerical values of $\hat{\rho}_0$, k_0, σ_0, and ε_0 at $t = t_0$ and of the relative compressive strain of the soil ε (1) and ε (20) for the soil creep at the instants of time t = 1 sec and t = 20 sec and $\hat{\rho}_0(0.05)$ at t = 10 sec in the cases of loading at $t \in [0; t_0]$ by the harmonic law (10) and by the linear law $\sigma = vt$ for different values of the initial density of the soil (when w = 19%).

Figures 2–4 give plots reflecting the results of certain experiments on investigating the creep of black earth and derno-podzolic soils. The dependences $\varepsilon(t)$ show that higher values of the initial soil density correspond to lower values of ε_0 for the same value of σ_0. The quantities ε_{st}, $\Delta\rho(h_0)$, and t_{st} decrease with growth in $\hat{\rho}$. As the loading rate increases at $t \in [0; t_0]$, the strain of the soil increases, whereas t_{st} decreases. An increase in σ'_t at $t \in [0; t_0]$ leads to a larger compaction of the soil. When the values of the compressive stresses are identical, the strains $\varepsilon(t)$ are more substantial in a moister soil. The values of ε_0, ε_{st}, $\Delta\rho(h_0)$, and t_{st} increase with soil humidity.

The stresses σ_0 increase with time t_0; therefore, the $\varepsilon(t)$ curves corresponding to different t_0 also characterize the influence of the value of σ_0 on the creep. The results obtained show that the increase in t_0 (and accordingly in σ_0) causes the quantities $\varepsilon(t)$ (at $t < t_{st}$), $\Delta\rho(h_0)$, and ε_{st} to grow.

We compared the results of the experiments on investigating the creep of derno-podzolic soil to the data of the corresponding experiments for black earth soil. A comparison of experimental results on determination of the influence of the initial density of the soil on $\varepsilon(t)$ and other characteristics has shown that the values of $\varepsilon(t)$, $\Delta\rho(h_0)$, ε_{st}, and t_{st} of the black earth soil are higher than those of the derno-podzolic soil for the same initial data. A comparison of the results of the experiments in which the influence of the loading rate on the indices under study was determined at $t \in [0; t_0]$ has enabled us to establish that the quantities ε_{st} and t_{st} for the black earth soil are larger than those for the derno-podzolic one for the same experimental data. It has been shown that the strains $\varepsilon(t)$ of the black earth soil at short t are smaller but thereafter they become much larger than those of the derno-podzolic soil as t grows. The black earth soil is compacted to a larger extent than the derno-podzolic soil, depending on both $\hat{\rho}$ and ω.

The loading rate at $t \in [0; t_0]$ exerts a significant influence on the rate of creep strain ε'_t at $t \in (0; \infty)$. The curves presented in Fig. 4 and other results obtained show that, as t increases, we have the rate $\varepsilon'_t \to 0$ for all values

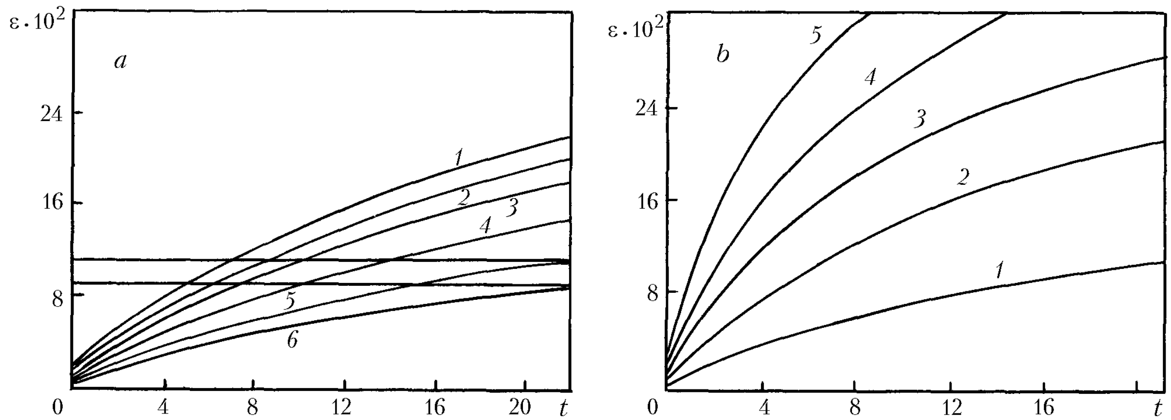

Fig. 2. Creep of black earth soil for different values of: a) the initial density of the soil [1) $\hat{\rho}$ = 1.1 and k = 1.7646, 2) 1.2 and 1.5686, 3) 1.3 and 1.3725, 4) 1.5 and 0.9804, 5) 1.7 and 0.5882, and 6) 1.9 g/cm^3 and 0.1960 g/(cm^3·m), ω = 3.15 sec^{-1}, σ_m = 36 kPa, t_0 = 0.1 sec, and w = 19%]; b) the rate of initial strain [1) ω = 0.9, 2) 2, 3) 3, 4) 4, and 5) 5.4 sec^{-1}, t_0 = 0.1 sec, w = 19%, $\hat{\rho}$ = 1.1 g/cm^3, and k = 1.7646 g/(cm^3·m)]. t, sec.

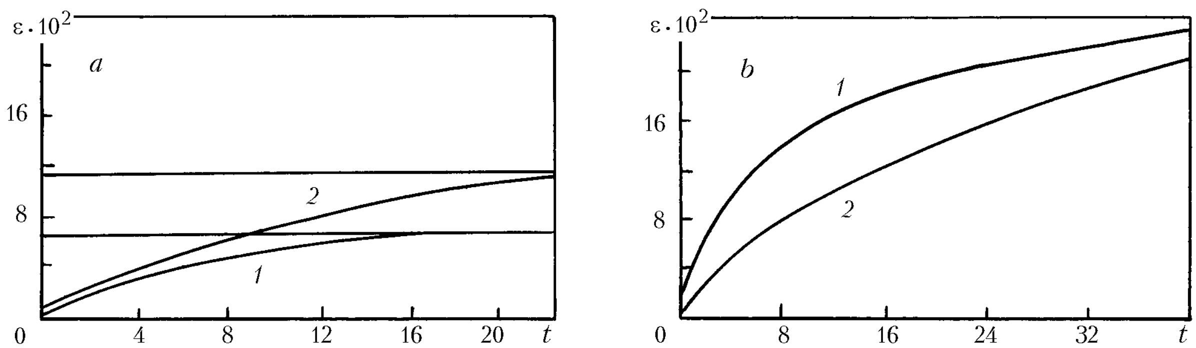

Fig. 3. Change in the relative compressive strain with time in creep of derno-podzolic and black earth soils in the case of the initial loading by the harmonic law: (1) in the derno-podzolic soil; 2) in the black earth soil): a) for $\hat{\rho}$ = 1.7 g/cm^3, k = 0.5882 g/(cm^3·m), ω = 3.15 sec^{-1}, t_0 = 0.1 sec, and w = 19%; b) for $\hat{\rho}$ = 1.1 g/cm^3, k = 1.7646 g/(cm^3·m), ω = 2.0 sec^{-1}, t_0 = 0.1 sec, and w = 19%. t, sec.

of ω. The higher the ω, the larger the ε_t' of both the derno-podzolic soil and the black earth soil. Under the same experimental conditions, the quantities ε_t' for the derno-podzolic soil are much larger than those for the black earth soil. To investigate the character and to quantitatively evaluate the influence of the initial density of the soil, the humidity, and the rate of strain or the initial loading at $t \in [0; t_0]$ on the relaxation properties in it in the case of simultaneous change in these factors we have performed computer calculations according to the programs developed; these calculations represent runs of full-scale, three-factor computer experiments. Two of them were carried out with the aim of investigating relaxation processes, whereas the other two were conducted to study creep processes. We considered the harmonic regimes of strain and loading (7) and (10) at $t \in [0; t_0]$ and the linear regimes.

The results of the full-scale factor experiments were used to obtain the correlation dependences of the compressive stresses in the soil, the relative and total compressive strains of the soil, and other indices on $\hat{\rho}_0$, w, ω, or v at different fixed instants of time $t \le t_{st}$. The coefficients of the regression equations and their regression (significance) and the adequacy of the regression equations were determined using the procedure of [13]. Testing according to Fisher's variance ratio for a 5% regression level has shown that the regression equations obtained are suitable for description of the experimental data. An analysis has revealed that almost all the indices under study are mainly affected

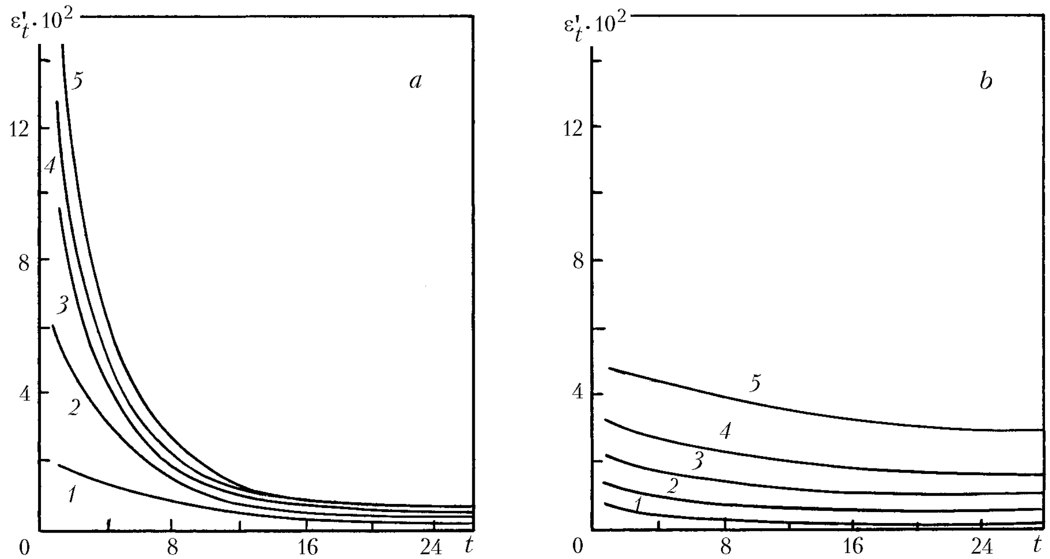

Fig. 4. Change in the rate of creep strain with time for different values of the rate of initial loading by the harmonic law: a) derno-podzolic soil; b) black earth soil [1) ω = 0.9, 2) 2, 3) 3, 4) 4, and 5) 5.4 sec^{-1}, t_0 = 0.1 sec, w = 19%, $\hat{\rho}$ = 1.1 g/cm^3, and k = 1.7646 g/(cm^3·m)]. t, sec.

by the initial density of the soil and the strain rate. In the full-scale factor experiments associated with investigation of relaxation stresses, the mean values of the quantities $\Delta\rho(h_0)$, $\hat{\rho}_0(0.05)$, σ_0, and $\sigma(t)$ at different t grow with $\hat{\rho}$ and strain rate. In the full-scale factor experiments associated with investigation of the soil creep, the mean values of ε_0, $\varepsilon(t)$, and $h(t)$ decrease as $\hat{\rho}$ increases and increase with ω.

The regression equations obtained show that the influence of the soil humidity in the considered range of its variation is substantially smaller than the influence of $\hat{\rho}$ and the strain rate at $t \in [0; t_0]$. It has been revealed that, as w grows, all the indices considered decrease in the relaxation processes and grow in the creep processes.

The above character of the influence of the initial density and humidity of the soil on its relaxation properties and compaction is determined by the following factor. The value of q grows with $\hat{\rho}$ ($q \to E$), whereas the characteristic g decreases ($g \to 0$). The properties of the soil approach elastic ones. The characteristic g grows with soil humidity w, whereas q decreases ($q \to 0$). The elasticity of the soil is reduced, and its properties approach viscous ones.

Measures intended to prevent the overcompaction of soils as a result of the action of agricultural equipment are required to solve the problem of conservation of the fertility. In a number of works, this action is evaluated not only by the values of compressive stresses arising in the soil in motion of vehicles [14]. The investigation carried out shows that there is no one-to-one correspondence of the compaction of the soil to the compressive stresses. The compaction of the soil is affected by a number of factors: the granulometric composition of the soil and the content of organic substances in it, its initial density, the humidity, the propagation depth of a deformable soil layer, the character and rate of change of compressive stresses and strains, the period of strain, and others. For substantiated evaluation of the change in the physical state and compaction of the soil it is necessary to take into account the mutual influence of all these factors.

The results (obtained in this work) of investigation of the relaxation processes in soils are consistent with the experimental results from works on relaxation processes in soils [2–4, 10, 12] and in other deformable media [5–7].

CONCLUSIONS

1. Based on the mathematical modeling of the viscoelastic properties of disperse media (including soils) by the differential equation (1), we have obtained analytical dependences and algorithms enabling us to find by calculation, with allowance for the time factor, the indices of the stressed-strained state of a medium with rheological properties varying with deformable-layer depth under different loading conditions.

2. We have investigated the processes of relaxation and creep of disperse media with rheological properties varying with deformable-layer depth; these investigations resulted in formulas and computational algorithms enabling us to determine the increment of the density of a medium at different depths and its hardening arising in loading of a deformable medium by different laws and in creep.

3. As a result of computer experiments, we have found the indices characterizing the creep and relaxation of black earth and derno-podzolic soils and their compaction and hardening under loading by different laws and in the process of creep.

4. We have revealed the correlation dependences of the indices investigated on the initial density of the soil, its humidity, and strain rate at $t \in [0; t_0]$. They enabled us to evaluate the influence of the most significant factors on the course of relaxation processes in the soil.

5. The investigation carried out shows that there is no one-to-one correspondence of the compaction of the soil to the compressive stresses. The compaction of the soil is influenced by a number of factors. For substantiated evaluation of the change in the stressed-strained state and compaction of the soil under the action of external load it is necessary to take into account the mutual influence of all these factors.

6. Close investigations of the rheological properties of soils of different types in different physical states are required. Revealing the governing rheological equations for different soils and the dependences of these parameters on the density, the humidity, and the strain rate will enable one to use refined computational methods of evaluation of the influence of negative anthropogenic actions on soils and to develop scientifically substantiated recommendations on reduction of these actions.

NOTATION

E, elastic modulus, MPa; g and g_0, transformed dimensionless characteristic of viscoelastic properties of the soil and its value on the soil surface at the beginning of the process of relaxation or creep; H, propagation depth of the deformable layer of the soil before its loading, m; H_{pr}, propagation depth of the compressive strain of the soil, m; H_0, propagation depth of the deformable soil layer at the beginning of the process of relaxation or creep, m; h and h_0, absolute compressive strain and its value at $t = t_0$, cm; k, angular coefficient of the (6) (straight) line, g/(cm^3·m); k_0, angular coefficient of the $\rho_0(\tilde{y})$ line at the beginning of the process of creep, g/(cm^3·m); p and q, parameters of the differential equation (6) for the soil (characteristics of viscoelastic properties of the soil), MPa and sec^{-1}; q_0, parameter of the governing differential equation (6) for the soil at $t = t_0$, MPa; t, time, sec; v, linear rate of change in strains or stresses in the case of their uniform change at $t \in [t; t_0]$; w, weight (absolute) humidity of the soil, %; x and z, axes in the coordinate system $Oxyz$; y, vertical coordinate of a particle of the deformable layer of the soil before its loading (depth), m; $\tilde{y}$, vertical coordinate of a particle of the deformable soil layer in the processes of relaxation and creep of the soil; Δ, increment of the quantity; ε, relative compressive strain; ε_{m}, amplitude of the relative compressive strain of the soil under strain by the harmonic law; ε_0, relative compressive strain of the soil at $t = t_0$; ρ and ρ_{dr}, densities of the moist and absolutely dry soils respectively, g/cm^3; $\overset{\frown}{\rho}$ and $\overset{\frown}{\rho}_0$, density of the soil before its loading at $y = 0$ (on the soil surface) and at the beginning of the process at $\tilde{y} = 0$, g/cm^3; σ, compressive stress, MPa; σ_{m}, amplitude of compressive stress in loading by the harmonic law; MPa; σ_0, compressive stress at $t = t_0$; σ_{str}, tensile strength of the soil, MPa; ω, frequency of the harmonic process of strain or loading, sec^{-1}. Subscripts and superscripts: m, maximum; N, point corresponding to the boundary of the segment $[0; N]$; s, point corresponding to the right-hand boundary of the small segment $[s; s-1]$ of the segment $[0; N]$; $'$, first derivative; str, strength; pr, propagation of strain; dr, dry soil; st, stabilization of strain.

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